INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL GRADUATE PHYSICS EXAMINATION 2015
Day & Date of Examination : Sunday, January 25, 2015
Time : 10 AM to 1 PM

Part A- Maximum Marks: 150   Time for Part A: 60 minutes

Part B- Maximum Marks: 150   Timefor Part B : 120 minutes

Solutions of part A

1. One of Maxwell's equations of E M theory is

   $\nabla . E = \dfrac{\rho}{\varepsilon_0}$ Substituting, $E = -Grad\, V = -\nabla V$,

   it gives $\nabla . (-\nabla V) = \dfrac{\rho}{\varepsilon_0} \Rightarrow \nabla^2 V = -\dfrac{\rho}{\varepsilon_0}$. This is

   Poisson's equation for potential V which in cartesian and spherical coordinates is written as (a) and (b).

   Ans: a, b & d

2. Rotational kinetic energy of a diatomic

   molecule is $= \dfrac{1}{2} I\omega^2$ which is expressed as

   $\dfrac{I^2\omega^2}{2I} = \dfrac{\left(\sqrt{J(J+1)}\,\hbar\right)^2}{2I}$ which is the format

   used in quantum mechanics

   Ans: b & d

3. A substance in superconducting state behaves as a perfect diamagnet.

   Ans: c

4. In 1914, a year before he was killed, the English physicist Henry Moseley found a relationship between X-ray wavelength an emitted by element and its atomic number. He was then able to re-sequence the periodic table by nuclear charge, rather than the atomic weight.

   Ans: b

5. Fine structures are the faint lines observed in the spectrum of hydrogen like atoms. These are explained by Sommerfeld elliptical model with relativistic correction and spin of electron as well as Spin Orbit interaction combined with relativistic correction.

   Ans: c & d

6. Using Claussius and Clapeyron equation

   $\dfrac{dP}{dT} = \dfrac{L}{T(V_2 - V_1)}$ and substituting

   $dT = -1°c, L = 4.2 \times 80 \times 10^{-3}$ joule / gram

   $T = 273K$ and $(V_2 - V_1) = (1 - 1.0191) \times 10^{-3}\, \text{m}^3$

   P + dP is calculated to be ≈ 136 atm.

   Ans: c

7. The mass of an electron moving with speed $\dfrac{c}{\sqrt{2}}$

   is calculated by $= \dfrac{m_0}{\sqrt{\left(1 - \dfrac{v^2}{c^2}\right)}} = m_0\sqrt{2}$ hence its

   momentum is $m_0\sqrt{2} \times \dfrac{c}{\sqrt{2}} = m_0 c$ and its de

   wavelength is $\lambda = \dfrac{h}{p} = \dfrac{h}{m_0 c}$ which is the

   same as Compton wavelength.

   Ans: a & b

8. Chlorine gas is often used as quenching agent in a Geiger Muller counter.

   Ans: d

9. In our modern understanding, strangeness is conserved during the **strong** and the **electromagnetic interactions**, but not during the **weak interactions.** The decay of neutral hyperon $\Lambda°$ is a weak interaction as it does not conserve strangeness.

   Ans: a

10. The orientational polarization of a dielectric is a temperature dependent phenomenon.

    Ans: c

11. See relevant standard text.

    Ans: a, b & d

12. The moment of inertia of the circular lamina is

$I = \int\limits_{0}^{R} 2\pi r^2 \, dr \, \rho_0 \left(1 - \frac{r}{R}\right) r^2.$ This yields $I = \frac{3}{10} MR^2$

Ans: b

13. For proton & neutron $m_P = m_N = m$ $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$= \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 0.08 \times 1.6 \times 10^{-19}}} = 1A°$

Ans: c & d

14. The given differential equation represents Forced oscillations.
Ans: d

15. An any thermodynamic process as $dQ = dU + PdV$

or $C \, dT = C_v \, dT + \int \frac{PV^n}{V^n} dV = C_v \, dT + K \int V^{-n} dV$

which gives $C = \frac{R}{\gamma - 1} - \frac{R}{n - 1}$

Ans: d

16. Two anti parallel currents, one above the other, repeal each other by a force $F = \frac{\mu_0}{2\pi} \frac{i_1 i_2}{D}$ N/m and

maintain equilibrium when $\frac{\mu_0}{2\pi} \frac{i_1 i_2}{D} = \lambda g$ where

λ is the mass per unit length. If the wire is

pressed a little, the force is $F = \frac{\mu_0}{2\pi} \frac{i_1 i_2}{(D-x)} = \lambda g \left(1 - \frac{x}{D}\right)^{-1}$

expanding binomially and neglecting higher power terms the net restoring force per unit

length is expressed as $\frac{\lambda g}{D} x$. Then the deflecting

force is equal to restoring force in equilibrium

So $\lambda \frac{d^2 x}{dt^2} = -\frac{\lambda g}{D} x$ which is the equation of

SHM whose time period $T = 2\pi \sqrt{\frac{D}{g}}$.

Ans: b

17. Power dissipated is $W = Vi \Rightarrow V = \frac{3.2}{2} = 1.6$ volt and

the bulk resistance $R_b = \frac{V}{I} = \frac{1.6}{2} = 0.8\Omega$

Ans: b & d

18. Electric field produced by the given non-conducting sphere at an outside point $(r \geq R)$ is

$E = \frac{1}{4\pi\varepsilon_0 r^2} \int\limits_{0}^{R} 4\pi\xi^2 d\xi \rho_0 \left(1 - \frac{\xi}{R}\right) = \frac{\rho_0 R^3}{12\varepsilon_0 r^2}$

The magnitude of electric field inside the sphere

as a function of r is $E = \frac{1}{4\pi\varepsilon_0 r^2} \int\limits_{0}^{r} 4\pi\xi^2 d\xi \rho_0 \left(1 - \frac{\xi}{R}\right)$

or $E = \frac{\rho_0 r}{3\varepsilon} \left(1 - \frac{3r}{4R}\right)$ Now setting $\frac{dE}{dr} = 0$ we get

$E_{max} = \frac{\rho_0 R}{3\varepsilon}$ at $r = \frac{2}{3} R$ Further the total

electrical energy stored in the non conducting sphere is obtained by

$U = \int\limits_{0}^{R} \frac{\varepsilon E^2}{2} \times 4\pi r^2 dr = 0.021 \frac{\rho_0^2 R^5}{\varepsilon}$

Ans: a, c & d

19. Idosulphate of quinine was first used by English Scientist Herapath as a dichroic material to obtain a Polaroid. Nowadays stretched films of polyvinyal alcohol impregnated with iodine are being used as Polaroid.
Ans: c & d

20. Poynting vector $S = \frac{E \times B}{\mu_0}$ is the energy flowing

per second per unit area of cross section. Thereby

$\varepsilon_0 \mu_0 \left(\frac{E \times B}{\mu_0}\right)$ has the dimension of

$\frac{1}{c^2} \left(\frac{watt}{m^2}\right) = \frac{Impulse}{m^3} = \frac{mountam}{volume}$

Ans: b & d

21. The average distance travelled by an excess carrier between its time of generation and recombination in a semiconductor defines diffusion length.

Ans: b

22. The angle of recoil (ϕ) in an event of Compton

scattering is expressed as $\tan \phi = \frac{\cot(\theta/2)}{1 + \frac{h\nu}{m_0 c^2}}$

showing that $\phi = \frac{\pi}{2}$ when $\theta = 0$ & $\phi = 0$ when $\theta = \pi$

Ans: c

23. Each of the two electrons revolving in opposite sense in s - orbit of hydrogen like atom experience a centripetal force $= m\omega^2 r$. When subjected to external magnetic field **B,** one of these electrons is accelerated while the other is

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retarded to give a resultant magnetic dipole moment $\Delta\mu = e\Delta\nu\pi R^2 = \dfrac{e^2 B\pi R^2}{4\pi m} = \dfrac{e^2 R^2 B}{4m}$

as $\Delta\upsilon = \pm\dfrac{\Delta\omega}{2\pi} = \pm\dfrac{eB}{2\pi m}$ is obtained from

$m(\omega + \Delta\omega)^2 r = m\omega^2 r \pm e\upsilon B$

Ans: b

24. Torsional rigidity of a solid shaft is expressed as $c = \dfrac{\pi\eta r^4}{2l} = \dfrac{\eta(\pi r^2)^2}{2\pi l}$

Ans: b & c

25. The parity of a system refers to the behavior of its wave function under inversion of coordinates through the origin. The parity operator has eigen values ± 1 (+1 means even parity while - 1 refers to odd parity). For hydrogen like atoms the parity is $P = (-1)^l$ which gives that the function has even parity if orbital angular momentum quantum number l is even and odd parity if l is odd.

Ans: a, b & d

Part B₁

B¹. Infinitesimal rotations are vectors where as finite rotations are not. It can be illustrated by showing that the addition is commutative i,e $d\theta_1 + d\theta_2 = d\theta_2 + d\theta_1$ for small angular rotations. However this expression does not hold true for finite rotations about different axes showing that finite rotations are not vectors.

B². Planck's Radiation formula is $u_\upsilon d\upsilon = \dfrac{8\pi h\upsilon^3}{c^3\left(e^{\frac{h\upsilon}{KT}}-1\right)}d\upsilon$

The total radiant energy

$E = \int_0^\infty \dfrac{c}{4}u_\upsilon d\upsilon = \dfrac{2\pi h}{c^2}\int_0^\infty \dfrac{\upsilon^3}{\left(e^{\frac{h\upsilon}{KT}}-1\right)}d\upsilon$ substituting

$\dfrac{h\upsilon}{KT} = x$ one obtains

$E = \dfrac{2\pi k^4 T^4}{h^3 c^2}\int_0^\infty \dfrac{x^3 dx}{e^x - 1} = \dfrac{2\pi k^4 T^4}{h^3 c^2}\dfrac{\pi^4}{15} = \sigma T^4$

B³. The average energy of Planck's oscillator is

$<E> = \dfrac{h\upsilon}{e^{\frac{h\upsilon}{KT}} - 1}$ which in the limit of $h\upsilon << kT$ turns out to be equal to kT, the classical value. Thus if Planck's constant h were smaller than its present value, results of quantum mechanics would not have been conspicuous than they are The statement is therefore refuted.

B⁴. If the two extremities of the line are designated as $(P_1 V_1)$ and $(P_2 V_2)$, the slope of the line will be $m = \dfrac{P_1 - P_2}{V_1 - V_2} = -\dfrac{P_1 - P_2}{V_2 - V_1}$. Taking a point

(PV) on the line, equation of line can be written as

$(P - P_1) = -\dfrac{P_1 - P_2}{V_2 - V_1}(V - V_1)$

changing the variable $P = \dfrac{RT}{V}$

$T = -\left(\dfrac{P_1 - P_2}{V_2 - V_1}\right)\dfrac{V^2}{R} + \left(\dfrac{P_1 - P_2}{V_2 - V_1}V_1 + P_1\right)\dfrac{V}{R}$

This is the equation of a parabola showing that T and hence the internal energy (u) increases first and then decreases when volume is increasesd.

B⁵. According to Einstein's mass energy relation when a photon of energy E ≥ 1.02 MeV is converted into matter to produce an electron positron pair in vacuum, the conservation of momentum yields the two particles to move with velocities ≥ c which is not acceptable.

B⁶. The self energy (work done in getting a charge assembled in its present form) of a spherical distribution of charge

$U = \dfrac{1}{4\pi\varepsilon_0}\int_0^R \dfrac{4\pi r^3 \rho}{3}4\pi r^2 dr\rho = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3q^2}{5R}$

increase with decreases in radius so more work. This justifies the statement.

B⁷. The given expression $i\hbar\dfrac{\partial}{\partial p}$ is an operator for position vector r in the momentum space.

B⁸. Mixing of two classical ideal gases is an irreversible process. Entropy always increases during an irreversible process.

B⁹. Reciprocal lattice is described by reciprocal vectors $A=\dfrac{b\times c}{(abc)}, B=\dfrac{c\times a}{(abc)} \& C=\dfrac{a\times b}{(abc)}$

where a, b & c are the lattice unit vectors.

In case of a bcc lattice with 'a' as the edge of unit cell, the primitive vectors may be expressed as

$\vec{a}=\dfrac{a}{2}(i+j+k),\ \vec{b}=\dfrac{a}{2}(i+j-k)\ \&\vec{c}=\dfrac{a}{2}(i-j+k)$

the corresponding reciprocal lattice vectors will then be

$\vec{A}=\dfrac{\vec{b}\times\vec{c}}{a.b\times c}=\dfrac{\dfrac{a}{2}(i+j-k)\times\dfrac{a}{2}(i-j+k)}{\dfrac{a^3}{2}}=-\dfrac{1}{a}(j+k)$

$\vec{B}=\dfrac{\vec{c}\times\vec{a}}{a.b\times c}=\dfrac{\dfrac{a}{2}(i-j+k)\times\dfrac{a}{2}(i+j+k)}{\dfrac{a^3}{2}}=-\dfrac{1}{a}(i-k)$

$\vec{C}=\dfrac{\vec{a}\times\vec{b}}{a.b\times c}=\dfrac{\dfrac{a}{2}(i+j+k)\times\dfrac{a}{2}(i+j-k)}{\dfrac{a^3}{2}}=-\dfrac{1}{a}(i-j)$

Thus these three reciprocal lattice vectors represent the primitive vectors of a fcc lattice. Hence the statement.

B¹⁰. The given circuit is a full wave voltage doubler. For half cycle one diode conducts and for another half cycle the another diode conducts yielding *twice* the peak of input voltage between the output terminals.

Part B₂

P¹. (a) When a particle of mass m moving with velocity u makes head on elastic collision with another particle of equal mass, the two merely exchange their velocities (This being a consequence of conservation of energy and momentum.) Thus with $u_1=u \& u_2=0$ we get $v_1=0$ and $v_2=u$

(b) In case the Kinetic energy after collision is three-fourth of original value then $u=v_1+v_2$ and $\dfrac{3}{4}u^2=v_1^2+v_2^2$ gives $v_2-v_1=\dfrac{u}{\sqrt{2}}$ Therefore the coefficient of restitution is $e=\dfrac{v_2-v_1}{u_1-u_2}=\dfrac{1}{\sqrt{2}}$

(c) The total energy after collision is

$\dfrac{1}{2}mv_1^2+\dfrac{1}{2}mv_2^2=E=0.2J$. Thereby

$v_1^2+v_2^2=\dfrac{2E}{m}$ Also $v_1+v_2=u$ substituting v_2

one obtains $v_1^2+(u-v_1)^2=\dfrac{2E}{m}$ or

$2v_1^2-2uv_1+u^2-\dfrac{2E}{m}=0$ Solving the quadratic

equation $v_1=\dfrac{2u\pm\sqrt{4u^2-4\times2\left(u^2-\dfrac{2E}{m}\right)}}{2\times2}$ For

v_1 to be a real value $u^2-2\left(u^2-\dfrac{2E}{m}\right)\geq0$

$\Rightarrow u^2\leq\dfrac{4E}{m}$ or $u\leq\sqrt{\dfrac{4E}{m}}$ or $u\leq\sqrt{8}=2\sqrt{2}$ m/S.

In an elastic collision however because of the conservation of energy $\dfrac{1}{2}mu^2=\dfrac{1}{2}mv^2=0.2J$

Thereby $u=\sqrt{\dfrac{2E}{m}}=2$ m/S Thus $2\leq u\leq2\sqrt{2}$ m/S

P². The gravitational attraction of two masses on 'm' is balanced at $-\dfrac{GM_A m}{x^2}=-\dfrac{GM_B m}{(6R-x)^2}\Rightarrow x=2R$

Hence to make an escape of mass m from body A, it must be given energy so as to take it to the

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gravitational front $x = 2R$. Means the required gain in PE must be equal to the initial kinetic energy So

$$\left[-\frac{GM_A m}{2R} - \frac{GM_B m}{4R}\right] - \left[-\frac{GM_A m}{R} - \frac{GM_B m}{5R}\right] = \frac{1}{2}mv^2$$

$$-\frac{G4\pi R^3 \rho m}{3(2R)} - \frac{G4\pi R^3 3\rho m}{3(4R)} - \left[-\frac{G4\pi R^3 \rho m}{3(R)} - \frac{G4\pi R^3 3\rho m}{3(5R)}\right] = \frac{1}{2}mv^2$$

$$v^2 = \frac{4}{5}\pi G\rho R^2 \Rightarrow v = 1.88 \text{ m/S}$$

P³.(a) According to Bohr Model the negatively charged muon will move in a circular orbit such that the necessary centripetal force is provided by electrostatic attraction i,e

$$\frac{m_\mu v^2}{r} = \frac{KZe^2}{r^2} \text{ or } m_\mu v^2 r = KZe^2 \text{ while Bohr}$$

quantum condition is $m_\mu vr = n\hbar$ now dividing

$v = \frac{KZe^2}{n\hbar}$ substituting the value, the radius is

obtained as $r = \frac{n^2\hbar^2}{m_\mu KZe^2} = \frac{n^2\hbar^2}{207mKZe^2} = 2.56 \times 10^{-13} \text{ m}$

(b) The average Kinetic energy of a molecule of gaseous hydrogen is

$$\frac{3}{2}kT = 13.6 \times 1.6 \times 10^{-19} \Rightarrow T = 1.05 \times 10^5 \text{ K}$$

P⁴. The condition of maxima for a grating is

$$(e+d)\sin\theta = n\lambda \qquad \text{......... (1)}$$

differentiating, the angular dispersion is

$$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta} = \frac{n}{\sqrt{(e+d)^2 - (e+d)^2 \sin^2\theta}}$$

or $\frac{d\theta}{d\lambda} = \frac{n}{\sqrt{(e+d)^2 - n^2\lambda^2}} = 3794.75 \text{ rad/m}$

while the linear dispersion is

$$f \times \frac{d\theta}{d\lambda} = \frac{fn}{\sqrt{(e+d)^2 - n^2\lambda^2}} = 948.7 \text{rad} \text{ where}$$

grating element $(e+d) = \frac{1}{6000} = 1.67 \times 10^{-6}$ m

Further the two wavelengths are diffracted at

$$(e+d)\sin\theta_1 = n\lambda_1 \Rightarrow \theta_1 = \sin^{-1}\left(\frac{n\lambda_1}{e+d}\right)$$

and

$$(e+d)\sin\theta_2 = n\lambda_2 \Rightarrow \theta_2 = \sin^{-1}\left(\frac{n\lambda_2}{(e+d)}\right)$$

Thus the linear dispersion of two wave lengths is

$\Delta y = f(\theta_1 - \theta_2)$ or

$$\Delta y = f\left[\sin^{-1}\frac{n\lambda_1}{e+d} - \sin^{-1}\frac{n\lambda_2}{e+d}\right] \text{ or}$$

$$\Delta y = f\left(\sin^{-1}0.70752 - \sin^{-1}0.70680\right)$$

$$\Delta y = 0.25(45.03 - 44.975) \times \frac{\pi}{180} = 0.24mm$$

Further the resolving power of the grating is

$$\frac{\lambda}{d\lambda} = nN \Rightarrow \frac{5890}{6} = 2N \text{ so that } N = 491 \text{ lines}$$

so the width of ruled space turns out to be

$$\Delta x = \frac{491}{6000} = 0.82mm$$

P⁵. Conservation of momentum in one dimensional collision between two particles which coalesces after collision yields

$$\frac{m_{01}u_1}{\sqrt{1 - \frac{u_1^2}{c^2}}} + m_{02} \times 0 = \frac{M_0 V}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad \text{--------(1)}$$

Energy considerations give

$$\frac{m_{01}c^2}{\sqrt{1 - \frac{u_1^2}{c^2}}} + m_{02} \times c^2 = \frac{M_0 c^2}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad \text{--------(2)}$$

Dividing equation (1) by (2)

$$\frac{m_{01}u_1}{m_{01} + m_{02}\sqrt{1 - \frac{u_1^2}{c^2}}} = V \qquad \text{--------(3)}$$

Substituting in (1) or

$$\frac{m_{01}u_1}{\sqrt{1 - \frac{u_1^2}{c^2}}} = \frac{M_0}{\sqrt{1 - \frac{V^2}{c^2}}} \times \frac{m_{01}u_1}{m_{01} + m_{02}\sqrt{1 - \frac{u_1^2}{c^2}}} \text{ or}$$

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$$\frac{m_{01}}{\sqrt{1-\frac{u_1^2}{c^2}}} + m_{02} = \frac{M_0}{\sqrt{1-\frac{m_{01}^2 u_1^2}{c^2\left(m_{01}+m_{02}\sqrt{1-\frac{u_1^2}{c^2}}\right)^2}}}$$

On squaring and rearranging one obtains

$$M_0^2 = m_{01}^2 + m_{02}^2 + \frac{2m_{01}m_{02}}{\sqrt{1-\frac{u_1^2}{c^2}}}$$ This is required.

P[6]. Internal energy of a gas is the energy possessed by it because of its molecular composition and may be defined as the sum of the kinetic and potential energies of all its molecular constituents. In case of an ideal gas the energy of all its molecules is purely kinetic According to Joule's law the internal energy of an ideal gas is a sole function of temperature and does not depend on pressure and volume i,e Ac

$$\left(\frac{dU}{dP}\right)_T = \left(\frac{\partial U}{\partial V}\right)_T = 0 \ \& \ \left(\frac{dU}{dT}\right)_P = \left(\frac{\partial U}{\partial T}\right)_V$$

Also U is a state function and dU is a perfect differential. First law of thermodynamics gives, $\Delta U = \Delta Q - \Delta W$. Unlike an ideal gas U for a real gas shares a potential energy term and

$$\left(\frac{\partial U}{\partial V}\right)_T \neq 0 \ \text{rather} \ U = \int C_V dT + \int \frac{a}{V^2} dv. \ \text{The}$$

behavior of a real gas however approximates to an ideal gas at low pressure & high temperature. Exchange of heat energy is a common process in thermodynamics and heat given or taken by a system is expressed as

$dQ = mcd\,T$ or $c = \frac{1}{m}\frac{dQ}{dT}$ denotes the specific heat. In case of a gas c defined by this expression brings in an ambiguity. To define c uniquely two types of specific heat of a gas are defined as $C_V = \left(\frac{dQ}{dT}\right)_V$ and $C_P = \left(\frac{dQ}{dT}\right)_P$ there by using second law of thermodynamics the difference is $C_P - C_V = T\left[\left(\frac{dS}{dT}\right)_P - \left(\frac{dS}{dT}\right)_V\right]$

considering the entropy S, a function of V and T

we write $dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$

dividing it by dT at constant pressure and

rearranging $\left(\frac{dS}{dT}\right)_P - \left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{dV}{dT}\right)_P$

substituting it back, one obtains

$$C_P - C_V = T\left[\left(\frac{\partial S}{\partial V}\right)_T \left(\frac{dV}{dT}\right)_P\right]$$ using Maxwell's

relation $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$ one gets

$$C_P - C_V = T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{dV}{dT}\right)_P$$ ------(1)

Now for one mole of an ideal gas $PV = RT$

gives $\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V}$ & $\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P}$ Thereby

$$C_P - C_V = T\frac{R}{V}\frac{R}{P} \Rightarrow C_P - C_V = R$$

For a Vander Waal gas $\left(P + \frac{a}{V^2}\right)(V-b) = RT$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V-b} \ \&$$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{(V-b)\left(\frac{RT}{(V-b)^2} - \left(\frac{2a}{V^3}\right)\right)}$$ substituting

in (1) $C_P - C_V = T\frac{R}{V-b} \frac{R}{(V-b)\left(\frac{RT}{(V-b)^2} - \left(\frac{2a}{V^3}\right)\right)}$

$$C_P - C_V = \frac{R}{\left(1 - \frac{2a(V-b)^2}{V^3 RT}\right)}$$ considering b to be

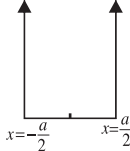very small $C_P - C_V = \frac{R}{\left(1 - \frac{2a}{VRT}\right)}$ or

$$C_P - C_V = R\left(1 - \frac{2a}{VRT}\right)^{-1} = R\left(1 + \frac{2a}{VRT}\right)$$

This gives the required expression.

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P⁷. One dimensional infinite potential well of width a centred at origin x=0 can be described

as $V(x)=0$ for $-\dfrac{a}{2} \le x \le \dfrac{a}{2}$

and $V(x)=\infty$ for $-\dfrac{a}{2} \ge x \ge \dfrac{a}{2}$

$x=-\dfrac{a}{2}$ $x=\dfrac{a}{2}$

The boundaries are

The one dimensional Schrodinger equation is

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2}+V(x)\Psi=E\Psi \qquad \text{------ (1)}$$

Within the infinite potential well, it becomes

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2}=E\Psi \ \ or \ \ \frac{d^2\Psi}{dx^2}+k^2\Psi=0 \ \text{----- (2)}$$

Where $k=+\sqrt{\dfrac{2mE}{\hbar^2}}$ A solution of differential

equation (2) may be $\Psi=A\sin kx+B\cos kx$ ----(3)

boundary conditions $\Psi=0$ at $x=\pm\dfrac{a}{2}$ implies

$0=-A\sin\dfrac{ka}{2}+B\cos\dfrac{ka}{2}$ at $x=-\dfrac{a}{2}$ ----(4) and

$0=A\sin\dfrac{ka}{2}+B\cos\dfrac{ka}{2}$ at $x=+\dfrac{a}{2}$ -------(5)

adding $2B\cos\dfrac{ka}{2}=0 \Rightarrow ka=(2n+1)\pi$

subtracting $2A\sin\dfrac{ka}{2}=0 \Rightarrow ka=(2n)\pi$

Hence the possible eigen values are $ka=n\pi$ or

$E=\dfrac{n^2\pi^2\hbar^2}{2ma^2}$ The first **three eigen values** being

$E_1=\dfrac{\pi^2\hbar^2}{2ma^2}$, $E_2=\dfrac{4\pi^2\hbar^2}{2ma^2}$, $E_3=\dfrac{9\pi^2\hbar^2}{2ma^2}$ The

corresponding eigen functions are respectively

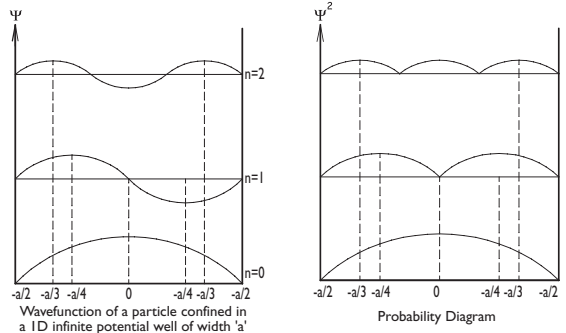$\Psi_{odd}=B\cos\dfrac{(2n+1)\pi x}{a}$ & $\Psi_{even}=A\sin\dfrac{2n\pi x}{a}$

where n = 0, 1, 2, 3, ….

$\Psi_1=B\cos\dfrac{\pi x}{a}$, $\Psi_2=A\sin\dfrac{2\pi x}{a}$ & $\Psi_3=B\cos\dfrac{3\pi x}{a}$

The normalization constant A is found as

$\int\Psi_n^*\Psi_n dx=1$ or $|A|^2\int\sin^2\dfrac{2n\pi x}{a}dx=1$

$|A|^2\int_{-a/2}^{a/2}\left(1-\cos 2\times\dfrac{2n\pi x}{a}\right)dx=1\Rightarrow A=\dfrac{1}{\sqrt{a}}$

in each case. The positions of maximum probability may now be find as

Wavefunction of a particle confined in a 1D infinite potential well of width 'a'

Probability Diagram

The expectation value may be obtained as

$\langle x\rangle=\int\Psi_n^* x\Psi_n dx=|A|^2\int x\sin^2\dfrac{2n\pi x}{a}dx$ or

$\langle x\rangle=|A|^2\int_{-a/2}^{a/2} x\left(1-\cos 2\times\dfrac{2n\pi x}{a}\right)dx$ or

$\langle x\rangle=|A|^2\int_{-a/2}^{a/2} xdx=0$ means the particle is

most likely to be at the origin x = 0 i,e at the middle of the well. Also

$\langle x^2\rangle=|A|^2\int_{-a/2}^{a/2} x^2\left(1-\cos 2\times\dfrac{2n\pi x}{a}\right)dx$

or $\langle x^2\rangle=|A|^2\int_{-a/2}^{a/2} x^2 dx=|A|^2\dfrac{a^3}{12}=\dfrac{a^2}{12}$

Thereby the uncertainty $\Delta x=\sqrt{\langle x^2\rangle-\langle x\rangle^2}$

$\Delta x=\sqrt{\dfrac{a^2}{12}-0}=\pm\dfrac{a}{2\sqrt{3}}$

P8. Zeeman splitting of spectral lines is

$\Delta\upsilon=\dfrac{eB}{4\pi m}$ Using $\upsilon=\dfrac{c}{\lambda}\Rightarrow d\upsilon=-\dfrac{c}{\lambda^2}d\lambda$

Then $\dfrac{c}{\lambda^2}d\lambda=\dfrac{eB}{4\pi m}\Rightarrow\dfrac{e}{m}=\dfrac{4\pi c}{\lambda^2 B}d\lambda$

$\dfrac{e}{m}=\dfrac{4\pi\times 3\times 10^8\times 0.161}{1\times 5890\times 5890\times 10^{-10}}=1.749\times 10^{11}\,C/Kg$

Resolving power of a prism of base thickness t

is $\dfrac{\lambda}{d\lambda}=t\dfrac{d\mu}{d\lambda}$ Thereby $\dfrac{5890}{0.161}=8888\,t\Rightarrow t=4.12$ cm.

P9. Maxwell's second equation $\nabla.B=0$ impresses upon the fact that **B** can be expressed as the

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curl of a vector potential A so we write it as $B = \nabla \times A = \text{curl } A$ so that $\nabla \cdot B = \nabla \cdot \nabla \times A$ = 0 (Since the divergence of a curl is always zero). This function **A** defines the magnetic vector potential which is useful to understand various aspects of electromagnetism. Fourth equation $\nabla \times B = \mu_0 J \Rightarrow \nabla \times (\nabla \times A) = \nabla (\nabla \cdot A) - \nabla^2 A = \mu_0 J$. Setting $\nabla \cdot A = 0$, one obtains $\nabla^2 A = \mu_0 J$.

The vector potential for a linear current element is often expressed as $A = \dfrac{\mu_0}{4\pi} \int \dfrac{i}{r} dl \dfrac{\mu_0}{4\pi} \int \dfrac{J}{r} dV.$

For a long straight conductor carrying current i, the vector potential $A = \dfrac{\mu_0}{4\pi} \int \dfrac{i}{r} \vec{dl}$

Using $r = \sqrt{x^2 + R^2}$ and $\vec{dl} = \eta dx$

$$A = \frac{\mu_0 i}{4\pi} \eta \int_{-L/2}^{L/2} \frac{dx}{\sqrt{x^2 + R^2}} \text{ or}$$

$$A = \frac{\mu_0 i}{4\pi} \eta \ln \left[x + \sqrt{x^2 + R^2} \right]_{-L/2}^{+L/2}$$

$$A = \frac{\mu_0 i}{4\pi} \hat{\eta} \ln \left[\frac{\dfrac{L}{2} + \sqrt{\dfrac{L^2}{4} + R^2}}{-\dfrac{L}{2} + \sqrt{\dfrac{L^2}{4} + R^2}} \right] \text{ or}$$

$$A = \frac{\mu_0 i}{4\pi} \hat{\eta} \ln \left[\frac{1 + \left(1 + \dfrac{4R^2}{L^2}\right)^{\frac{1}{2}}}{-1 + \left(1 + \dfrac{4R^2}{L^2}\right)^{\frac{1}{2}}} \right] \text{ For } L \gg R$$

$$A = \frac{\mu_0 i}{4\pi} \hat{\eta} \ln \left[\frac{1 + \left(1 + \dfrac{1}{2}\dfrac{4R^2}{L^2}\right)}{-1 + \left(1 + \dfrac{1}{2}\dfrac{4R^2}{L^2}\right)} \right] = \frac{\mu_0 i}{4\pi} \hat{\eta} \ln \left(1 + \frac{L^2}{R^2}\right)$$

$A = \dfrac{\mu_0 i}{4\pi} 2\hat{\eta} \ln \left(\dfrac{L}{R}\right).$ This gives vector potential for a current carrying straight wire. Considering two parallel wires carrying equal and opposite current, the vector potential A is expressed as

$$A = \frac{\mu_0 i}{4\pi} 2\hat{\eta} \left\{ \ln\left(\frac{L}{R_1}\right) - \ln\left(\frac{L}{R_2}\right) \right\} = \frac{\mu_0 i}{4\pi} 2\hat{\eta} \ln\left(\frac{R_2}{R_1}\right)$$

P10. From KVL (in first fig.):

$V_{CC} - I_B R_B - V_{BE} - I_E R_E = 0$ Putting

$I_E = I_C + I_B = \beta I_B + I_B = (1+\beta) I_B$

$V_{CC} - V_{BE} = (1+\beta) I_B R_E + I_B R_B$

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta+1) R_E} = \frac{12 - 0.7}{\{220 + (100+1) \times 3.3\} \times 10^3}$$

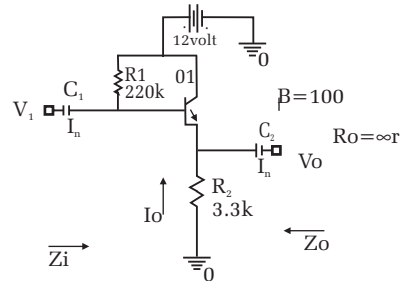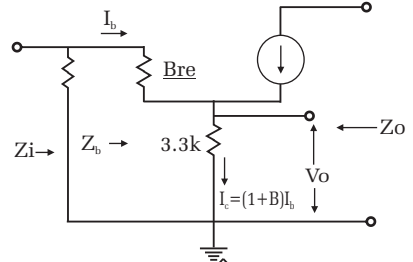$\therefore I_B = 20.42 \mu A$

Further

$I_E = (\beta+1) I_B = (100+1) \times 20.42 = 2.062 mA$

Then $r_e = \dfrac{26mV}{I_E} = \dfrac{26mV}{2.062mA} = 12.61 \ \Omega$

Now

Equivalent Circuit

$Z_b = \beta r_e + (\beta+1) R_E = 100 \times 12.61 + 101 \times 3300$

$Z_b = 334.56 \, k\Omega \cong \beta R_E$ and $Z_i = R_B \parallel Z_b$

$$Z_i = \frac{220 \times 334.56}{220 + 334.56} \times 10^3 = 132.72 k\Omega$$

$Z_o = R_E \parallel r_e$ or

$Z_o = \dfrac{3300 \times 12.61}{3300 + 12.61} = 12.56 \ \Omega \cong r_e$ Finally the

voltage gain $A_V = \dfrac{V_o}{V_i} = \dfrac{R_E}{R_E + r_e} = \dfrac{3300}{3300 + 12.61}$

or $A_V = 0.996 \cong 1$

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